

Feasibility of a metamagnetic transition in correlated systems

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The long-standing issue of the competition between the magnetic field and the Kondo effect, favoring, respectively, triplet and singlet ground states is addressed using a cluster slave-rotor mean field theory for the Hubbard model and its spin-correlated, spin-frustrated extensions in 2 dimension. The metamagnetic jump is established and compared with earlier results of dynamical mean-field theory. The present approach also reproduces the emergent super-exchange energy scale in the insulating side. A scaling is found for the critical Zeeman field in terms of the intrinsic coherence scale just below the metal-insulator transition where the critical spin fluctuations are soft. The conditions for metamagnetism to appear at a reasonable field are also underlined. The Gutzwiller analysis on the 2D Hubbard model and a quantum Monte Carlo calculation on the Heisenberg spin system are performed to check the limiting cases of the cluster slave-rotor results for the Hubbard model. Low-field scaling features for magnetization are discussed.

PACS numbers: 75.30.Kz, 71.10.Hf, 71.30.+h, 64.60.F-, 71.27.+a

I. INTRODUCTION

The question of metamagnetism in strongly correlated systems has been a long-studied subject. In the absence of applied magnetic field, Hubbard model shows a metal-insulator transition (MIT) driven by local correlation or doping^{1,2}. The physics close to the MIT is likened to the formation and subsequent quenching of the moments in the Kondo impurity model³. The connection between the two is borne out in the dynamical mean-field theory where the correlated lattice model is mapped on to an impurity model, exact in infinite dimension. The quenching of the local spin fluctuations leads to Fermi liquid (FL) behavior at low temperatures, as shown in⁴. This self-consistent emergence of a low energy coherence is typical of strongly correlated systems. Close to a Mott transition, such systems show a residual AF exchange between local moments. How an external magnetic field interacts with the moments, especially in the correlated metallic regime where spin fluctuations are still extant, is a question of considerable interest. The experimental observation of metamagnetic transition (MMT) in single crystal of bilayer perovskite metal $\text{Sr}_3\text{Ru}_2\text{O}_7$ ^{7,8}, an insulating magnet BiMn_2O_5 ^{9,10}, multiferroic hexagonal insulator HoMnO_3 ¹¹, and heavy fermions like MnSi ¹², CeRu_2Si_2 ¹³ rekindled the interest recently.

Gutzwiller approximation-based approaches^{5,6} motivated by the question of MMT in He^3 showed that there indeed is an MMT beyond a critical correlation (lower than the critical Hubbard- U for MIT). The presence of an MMT is, however, contradicted by Weigers¹⁴ et al. who found a smooth variation of magnetization with applied field, in tune with Stoner's¹⁵⁻¹⁸ approach. However, Stoner's theory, essentially a high temperature approach, underestimates the local correlation and misses the competition between the local moments and the magnetic field. Gutzwiller approximation, on the other hand⁵, is

incapable of describing the ground state of the correlated metallic phase properly. However, it is a theory for the ground state only and neglects spin correlations entirely. Theoretical attempts^{2,19,20} have recently been brought to bear upon this problem recently to understand it using more powerful techniques. The emergence of dynamical mean-field theory (DMFT) has seen a major paradigm shift in the study of strongly correlated systems. These calculations reveal the presence of MMT in a half-filled Hubbard model. As DMFT captures the dynamics close to the transition in great detail² and treats the local spin fluctuations better, the corresponding low energy scale naturally emerges. In the weakly correlated metal, a smooth transition is observed from an unpolarized metal to a polarized band insulator. In the strong coupling limit (close to U_c), the phenomenon is distinctly different though – showing a metamagnetic jump which drives the system from a strongly correlated metal to a field-driven band insulator. DMFT, however, could be numerically more intensive, depending on the choice of the impurity solver: moreover, the cluster extension of DMFT (C-DMFT) and the retrieval of spin-fluctuation energy scale in the insulating side are fairly demanding tasks.

While single-site DMFT and C-DMFT are perhaps some of the most efficient techniques for correlated systems, they have their own frailties in describing the insulating state with magnetic order. Incorporating spin correlations in these approaches is non-trivial. In addition, strongly correlated systems often come up with situations where spin and charge of an electron appear to behave distinctly and their responses to external probes are quite disparate. Such situations are missed in DMFT-based theories. A natural separation of these two distinct degrees is the key to slave-rotor (SR) approach. The Hilbert space of the physical electron is decoupled into the so-called “chargon” (conjugate to rotor) and “spinon”

spaces and the unphysical states are eliminated via local constraints. The strongly correlated problem then maps on to interacting slave particles self-consistently coupled to a gauge field. The gauge fluctuations, being weak, provide a framework for studying the Mott-Hubbard physics at intermediate to large coupling^{23,24} in a straightforward manner.

The present work uses slave-rotor mean-field (SRMF) theory to investigate the MMT within the Hubbard-Heisenberg model. In what follows we consider only the Zeeman field, as the orbital contribution is much weaker. We not only find the MMT but also address the question: *why does metamagnetism (MM), arising out of competition between Zeeman field and Kondo fluctuation, remain so elusive experimentally?* We predict a scaling behavior for the critical Zeeman field in terms of the Kondo scale, in the strongly coupled metallic regime. A possible experimental realisation of the transition in Mott-Hubbard systems is prescribed as well. We show that tuning the system to strong coupling (via tensile strain, for example) can act as a precursor for field-driven MMT. We identify the regimes for the observation of MM at the emergent ‘spin-exchange’ energy scale in the insulating side. Possible experimental realization of MM in real materials or optical lattices, out of correlation and external driving field, is the primary focus of the present work. Using extensive qualitative and quantitative arguments and a standard semi-analytical technique, we analyze the possible emergence of MM in correlated electronic systems. Indeed, there are various other slave-particle mean-field techniques used in the context of the Hubbard model^{28,31}. As we see below, the SRMF theory gives good results at a nominal numerical cost particularly in the strong coupling limit, where MM is most likely to be observed.

II. MODEL AND FORMALISM

We consider the $t-t'-U-J$ model on a square lattice in the presence of external magnetic field. This is a general correlated model without particle-hole symmetry at half-filling, and AF spin-exchange built in. The various models studied below are different limiting cases of this, discussed as we go along.

$$H = - \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + J \sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_\sigma \sigma n_{i\sigma}. \quad (1)$$

where $t_{ij} = t, t'$ are the nearest and next nearest neighbor hopping amplitudes respectively. $c_{i\sigma}^\dagger (c_{i\sigma})$ is electron creation (annihilation) operator at a given site. $n_{i\uparrow} (n_{i\downarrow})$ is the density operator for the up (down) spin. $J(>0)$ introduces antiferromagnetic (AF) spin exchange between spins at neighboring sites, while h is the external Zeeman field. In terms of rotor and spinon operators, this

model can be written as (see Florens, et al.²³ for SRMF formulation)

$$H_{SR} = - \sum_{i,j,\sigma} t_{ij} f_{i\sigma}^\dagger f_{j\sigma} e^{-i\theta_i} e^{i\theta_j} + U/2 \sum_i n_i^\theta (n_i^\theta - 1) + J \sum_{i,j} \mathbf{S}_i^f \cdot \mathbf{S}_j^f \quad (2)$$

$f_{i\sigma}^\dagger$ is the spinon creation operator, and the rotor creation (annihilation) operator is $e^{i\theta_i}$ ($e^{-i\theta_i}$). n_i^θ is chargon density operator and \mathbf{S}_i^f is $f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{j\beta}$. In the SRMF approximation the local constraint is relegated to a global constraint satisfied on the average.

$$\langle n_i^\theta \rangle + \langle n_{i,\uparrow}^f \rangle + \langle n_{i,\downarrow}^f \rangle = 1. \quad (3)$$

$\langle n_i^\theta \rangle$ and $\langle n_i^f \rangle$ are the average chargon and average spinon density respectively. Following straightforward algebra, the Hamiltonian Eq.2 decouples into two coupled Hamiltonians solved self-consistently under the saddle-point approximation.

$$H_f = - \sum_{i,j,\sigma} t_{ij} B_{ij} f_{i\sigma}^\dagger f_{j\sigma} + J \sum_{i,j} \mathbf{S}_i^f \cdot \mathbf{S}_j^f - \sum_{i\sigma} (\mu_f + h\sigma) n_{i\sigma}^f \quad (4)$$

$$H_\theta = - 2 \sum_{i,j,\sigma} t_{ij} \chi_{ij} e^{-i\theta_i} e^{i\theta_j} + U/2 \sum_i (n_i^\theta)^2 - \mu_\theta \sum_{i\sigma} n_i^\theta \quad (5)$$

where $B_{ij} = \langle e^{-i\theta_i} e^{i\theta_j} \rangle_\theta$ and $\chi_{ij} = \langle f_{i\sigma}^\dagger f_{j\sigma} \rangle_f$. μ_f, μ_θ are Lagrange multipliers for the number constraint: two multipliers are generally used for convenience to control $\langle n_i^\theta \rangle$ and $\langle n_i^f \rangle$ separately while still satisfying Eqn.(3). In the presence of spin density wave (SDW) with commensurate ordering wave vector $\mathbf{Q}=(\pi, \pi)$, the mean-field spinon Hamiltonian is

$$H_f^{MF} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k},\sigma} f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} - 2Jm \sum_{\mathbf{k}} (f_{\mathbf{k}\uparrow}^\dagger f_{\mathbf{k}+\mathbf{Q}\uparrow} - f_{\mathbf{k}\downarrow}^\dagger f_{\mathbf{k}+\mathbf{Q}\downarrow}) \quad (6)$$

where,

$$\epsilon_{\mathbf{k}\sigma} = - 2(tB + 3J\chi/4)(\cos k_x + \cos k_y) - 4t' B' \cos k_x \cos k_y - \mu_f - h\sigma \quad (7)$$

and $\lambda_{\mathbf{k}\sigma}^\pm = \frac{1}{2}(\epsilon_{\mathbf{k}\sigma} + \epsilon_{\mathbf{k}+\mathbf{Q}\sigma}) + \frac{1}{2}E_{\mathbf{k}}$; $E_{\mathbf{k}} = [(\epsilon_{\mathbf{k}\sigma} - \epsilon_{\mathbf{k}+\mathbf{Q}\sigma})^2 + (4Jm)^2]^{\frac{1}{2}}$. At half-filling, the corresponding self-consistency equations for the spinon sector are easily obtained and the magnetization is

$$M = \frac{1}{2} \sum_{\mathbf{k},\sigma} \sigma (n_F(-\lambda_{\mathbf{k}\sigma}^-) + n_F(-\lambda_{\mathbf{k}\sigma}^+)) \quad (8)$$

χ, χ' are, respectively, spinon kinetic energies for nearest and next nearest neighbor hoppings and m is staggered magnetization.

A. Single site theory

The simplest version of SRMFT involves decoupling at the single site level, neglecting intersite correlations in the mean-field. The single-site mean-field Hamiltonian for the rotor sector is

$$H_\theta = -8(t\chi + t'\chi')\phi(e^{-i\theta} + e^{i\theta}) + U/2(n^\theta)^2 - \mu_\theta n^\theta \quad (9)$$

In the present approximation $B = B' = \phi^2$. This rotor kinetic energy acts as the order parameter-quasiparticle (QP) weight (Z) for the fermions. If ϕ^2 vanishes, the system is driven into a Mott insulating state with no charge fluctuations. In that case, clearly, once ϕ is zero the effect of Coulomb correlation on the spinon part also vanishes. It is, therefore, necessary to take account of the intersite correlations to study magnetic interactions in the strongly correlated regime.

B. Two site (cluster) theory

We extend the theory to the bond (cluster) approximation in view of the shortcomings of the single-site theory. Here B' should be different from B - the nearest and next nearest neighbour correlations in charge sector are different. The rotor Hamiltonian for the two-site cluster is

$$H_\theta = -2t\chi(e^{-i\theta_1}e^{i\theta_2} + h.c.) + (6t\chi + 8t'\chi')\phi(e^{-i\theta_1} + e^{-i\theta_2} + h.c.) + U/2(n_1^\theta)^2 + U/2(n_2^\theta)^2 - \mu_\theta(n_1^\theta + n_2^\theta) \quad (10)$$

This Hamiltonian is again diagonalized numerically in the basis $|n_1^\theta, n_2^\theta\rangle$, where $B' = \phi^2$ with $\phi = \langle e^{\pm i\theta} \rangle$ and $B = \langle e^{-i\theta_1} e^{i\theta_2} \rangle$. When ϕ goes to 0 (i.e., the insulating phase), the nearest neighbour inter-site correlation, absent in the single-site case, could assume a non-vanishing value in the cluster approximation. The first term in equation (10) gives a finite rotor kinetic energy ($\sim t^2/U$), which, in turn, affects the spinon sector and makes the otherwise sterile $\phi = 0$ phase interesting. A bond approximation approach, therefore, is capable of recovering the inter-site spin correlation scale.

III. RESULTS AND DISCUSSIONS

To begin with, we briefly discuss our results on the Hubbard model at half-filling at $T = 0$ (all energies are given in units of t) and identify the Mott transition in SRMF. Results on MMT in this limit exists in DMFT^{2,20} and a comparison of the same using SRMF theory is therefore in order. No result on MMT is available on this model using SRMF theory (or any other slave-particle methods) so far. We begin by studying single-site and

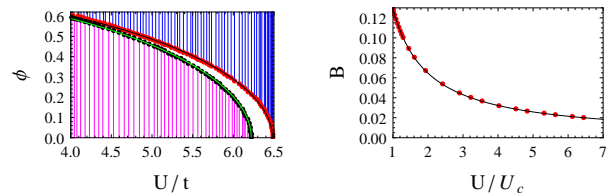


FIG. 1. (Color Online) Left figure: ϕ is plotted against U/t for single-site (red dots are data points and the black continuous curve is a fit to Brinkman-Rice picture) and cluster approximation (black dots are data points and the continuous green line is the Brinkman-Rice fit). The critical value of U/t for MIT is, 6.483 and 6.214 in single-site and cluster analysis respectively. Right figure: The inter-site correlation from the cluster analysis in the insulating phase (red dots) and the fit to t^2/U (black curve).

cluster approach for the Hubbard Hamiltonian with and without field respectively. We also observe how the spin-spin exchange interferes with correlation and how the spin fluctuations are affected in the proximity of MIT.

A. Hubbard ($t - U$) model on a square lattice: MIT and metamagnetism

It is well-known that in the $t - U$ model the local correlation drives the paramagnetic metal to a non-magnetic insulator in a Brinkman-Rice like transition (the order parameter ϕ smoothly going to zero (Fig. 1, left)). The divergence of the effective mass is signalled by vanishing of quasi-particle weight ($Z = \phi^2$) at the critical point. Although the nature of the insulating or metallic phase remains non-magnetic (unless we resort to a two-sublattice formalism similar to that of DMFT), irrespective of site or cluster analysis in the SR calculations, we note a different critical value of U/t for MIT in the site and cluster SRMF approach. As spatial correlations and non-local phase fluctuations are accounted for in the cluster, the critical U/t for MIT in a cluster approach is expected to be lower. The parameter (B) quantifying non-local fluctuations remains finite in the putative insulating phase and approaches zero in the $U/t \rightarrow \infty$ limit. For single-site analysis, we find $U_{c,site} = 6.483t$ while a two-site cluster gives a critical $U_{c,bond} = 6.214t$ for MIT. As far as the reduction in U_c is concerned, B plays much the same role as a non-local spin-spin correlation. In the insulating phase, B is non-zero (Fig. 1, right) and effective hopping remains finite, i.e., the effective mass does not diverge at MIT. In the cluster extension of the theory, therefore, spinons have a Fermi surface with finite Luttinger volume²⁹ even in the insulating phase.

We note that the value $U_{c,cluster}$ for MIT that we find is in good agreement with the C-DMFT prediction²² of $U_{c2} = 6.050t$ ³⁰. However, the AF nature of the insulating ground state of the half-filled Hubbard model remains beyond the scope of SRMF analysis. The paramagnetic

insulator in the SRMF approximation is an artefact of the assumption that the correlation acts in the rotor sector only, i.e., on the charge degrees alone; spins are free, having only a renormalized effective mass. There is no a priori reason, therefore, why charge ordering would lead to spin ordering. As a consequence the SRMF approach works better for systems with strong magnetic frustration, where a spin liquid insulator is likely. Both the single site and cluster Mott transitions, have a Brinkman-Rice nature. The quasiparticle weight goes to zero at critical Hubbard U (U_c) in a continuous fashion, and Z scales perfectly with $\sim 1 - (U/U_c)^2$ (Fig. 1-right panel).

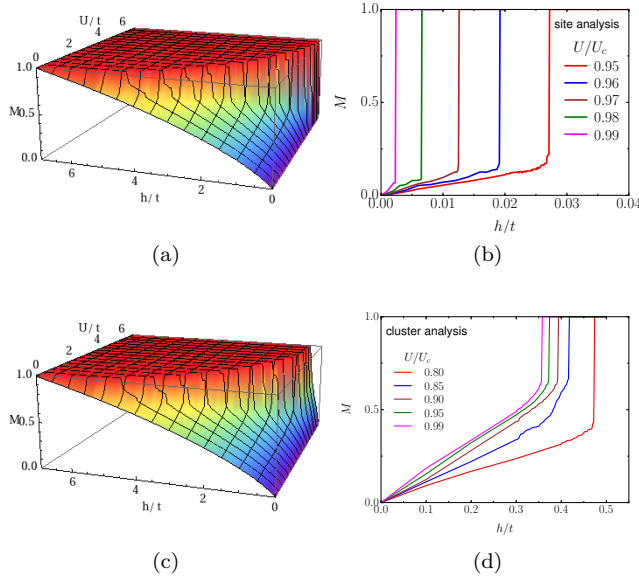


FIG. 2. Variation of magnetization showing the MMT in site [(a), (b)] and cluster [(c), (d)] approximation. $M - h/t$ plots close to the transition region of site and cluster theory are separately shown in (b) and (d) respectively.

1. Nature of MMT: Single-site analysis

How the system reacts to an external magnetic field, for the whole range of U (up to $U_{c,site}$), is shown in the phase diagram (Fig. 2(a)). For any finite U , ferromagnetism shows a first order jump to its saturation value at some critical field, clearly an MMT, instead of a smooth enhancement to magnetic saturation (as predicted originally by Stoner and extended later by Weigers et al.¹⁴ using spin fluctuation theory). This abrupt jump in magnetization leads to an MIT, the order parameter going to zero ($\phi = 0$) in a highly discontinuous manner at the same instant. This is a field-driven first order transition, rather than correlation-driven. The field moves the up and down spin bands apart leading to a weakly correlated, polarized band insulator²⁰. The closer one approaches the critical Coulomb repulsion, it becomes more susceptible to such a transition at a lesser field

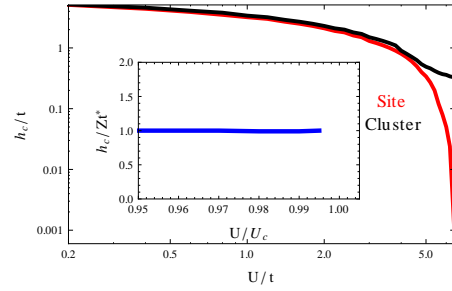


FIG. 3. h_c/t in log scale against U/t to highlight the huge scale variations for the critical numbers in the site and cluster analysis (significantly different, not apparent from the phase diagram in Fig.2). Inset shows h_c , scaled with the renormalized hopping, is constant with renormalized correlations.

(Fig. 2(b)). Physically, at such large values of U , the kinetic degrees of freedom are nearly quenched, effective mass is large - the strongly correlated metal is now susceptible to MIT. The presence of MM is confirmed for every finite $U < U_c$.

2. Nature of MMT: A two-site cluster analysis

In the single-site theory, the insulating ground state becomes ferromagnetic for an infinitesimal Zeeman field. The rotor Hamiltonian being local, there is no magnetic exchange scale. This is a well-known pathology of the single-site approximation in SR. In principle, one should look for the competing dynamics between an aligning field and spin-fluctuation, recovered in the cluster version of the theory (Fig. 2(c)). Two diagrams, site and cluster, are plotted (Fig. 2(b),(d)) to showcase the difference between the two schemes. The MMT in the metallic state is almost similar in the two cases – for any U there is a transition. In the insulating state however, any infinitesimal h causes saturation in magnetization (M) in the single-site case, while a finite critical magnetic field (Fig. 3, main panel), representing the emergent AF spin-correlation scale t^2/U (Fig. 1-right panel) is required for the cluster. The putative insulating state, where ϕ becomes zero, has interesting extant dynamics via inter-site correlation B within the cluster approximation; the rotor is still coupled to the spins and the renormalized kinetic energies are finite in the insulating phase. In both cases, therefore, the vanishing of the spin-stiffness (χ) signifies the metamagnetic jump: in the insulating region $h_c/t = 0$ in single-site case while for the cluster, h_c/t has to be finite (Fig. 3, main panel) to overcome the spin-correlation scale.

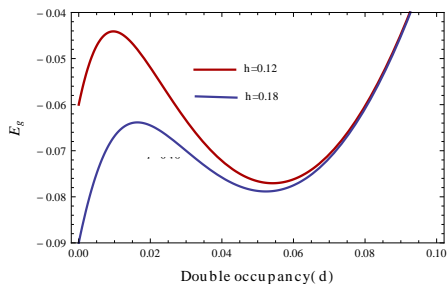


FIG. 4. (Color Online) Ground state energy (E_g) of the Hubbard model in Gutzwiller approximation as a function of double occupancy d at $U/U_c = 0.75$ on a square lattice for different h (0.12, 0.18), clearly showing the first order transition.

3. Scaling behavior for critical Zeeman field

We note that very close to the transition, for correlated models $\frac{h_c(U)}{Zt^*}$ becomes independent of U . A strong Hubbard correlation of order $0.97U_c$ to $0.995U_c$ shows that this ratio becomes a constant (Fig. 3, inset); and it is only within this range that h_c has experimentally feasible values. It is interesting to note that, Zt^* measures the effective renormalized bandwidth of the dispersive quasi-particles. The strong coupling limit of the problem renormalizes this number to a significantly smaller one, a measure of the diminishing coherence of the quasi-particles. The h_c in the strong coupling limit is the field required to destroy the coherence and favor a triplet spin state. Beyond this scale, the spin dynamics is determined only by h_c , thereby making $\frac{h_c(U)}{Zt^*}$ universal, irrespective of the value of U . We find similar scaling feature for h_c in higher dimensional bi-partite lattices. The scaling should hold true for bi-partite lattices in any dimension as the hopping is renormalized by \sqrt{d} factor ($t^* = t/\sqrt{d}$), where d is the dimension of the lattice. Hence the scaling is related to the competition between the coherence of the quasiparticles and the external agent (field) trying to destroy the coherence and is independent of the dimensionality of the non-interacting bath.

4. Comparison with single-site DMFT, Gutzwiller approximation and quantum Monte Carlo (QMC)

In earlier studies of MMT^{2,5,20}, the value of the field (h) was limited below the hopping parameter t . In such a situation, they looked for the value of U for which an MMT can be observed. The SRMF approach gives a critical field for MMT at any finite U . For $h < t$, we can compare our results with the previous work. For $T=0$, we compare the value of U/U_c below which there is no MMT (as h rises to t): our single-site SRMF theory gives a value close to 0.5 while in GA it is about 0.44⁵ and in DMFT, about 0.61^{2,20}. We also compare GA with SRMF. Vollhardt⁵ studied the effect of out of plane Zeeman field on the Hubbard model with a flat band using

GA. While we choose to do the same for a square lattice dispersion, an exact analytical solution is not possible in this case. The value of U_c for Hubbard model on a square lattice is about 6.451 in GA and with single-site SRMFT we find it to be 6.483. The agreement between these two results motivates us to verify the critical value of field (h_c) that induces a metamagnetic jump for a given U within GA. This can be evaluated in two ways from GA: plot magnetization versus field and search for the metamagnetic point, or find the point of flipping of the absolute minimum in the ground state energy (E_g). The E_g in GA is calculated for square lattice semi-analytically and it is a function of double occupancy (d), magnetization (M) and applied field (h). On minimization of energy an expression for m as a function of h and d is obtained. Putting it back in E_g , the minimum is located numerically. The plot of E_g against d shows two minima, one at zero and the other at a finite value of d (Fig. 4). The minimum at non-zero d remains the absolute minimum up to some critical field. At a certain h_c the absolute minimum flips from a finite value to $d = 0$ (Fig. 4). For $U/U_c = 0.75$, the critical value of field comes out to about 0.180. The value obtained from the SR analysis under the same set of parameters is 0.20, in reasonably good agreement with GA. This similarity in the relevant scales emerging out of GA and SRMFT is expected as both of them work well in the strong coupling limit. Fi-

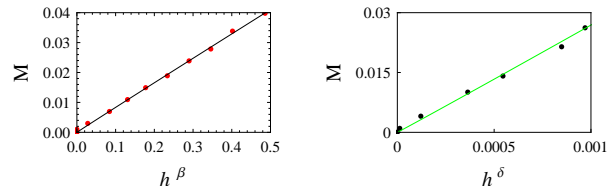


FIG. 5. Scaling behavior of the magnetization in a 2D square lattice for the Heisenberg model (left) and the Hubbard model (at $U/U_c = 0.99$, right figure). The two exponents, β and δ are found to be 1.02 and 0.98, where M is found to scale linearly with h^β and h^δ .

nally we perform a QMC analysis on an AF Heisenberg spin model on a square lattice, in presence of perpendicular Zeeman field. On increasing the lattice sizes, we check that the $M - h$ response for the system remains nearly the same for $L = 24, 32$ and 40 (where L^2 is the lattice size). We find the exponent (β)³² for magnetization against field for the $L = 32$ system and compare the number with the exponent (δ) (Fig. 5) in the large- U limit of the Hubbard model. The exponents seem to agree well, which is expected since Heisenberg model is the Large- U limit of the Hubbard model at half-filling. Though the numerically found exponents from the two techniques are slightly different from one (1.02 and 0.98), within our numerical accuracy, it could well be that they are, in reality, just 1.0 in the low field limit.

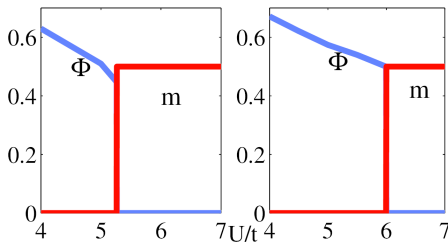


FIG. 6. (Color Online) Similarity of the phase diagram of (a) $t - t' - U - J$ model on the square lattice and (b) $t - U - J$ model on a triangular lattice. ϕ is the order parameter and m , the staggered magnetization ($J = 0.25$, $t' = t/2$).

B. $t - U - J$ model on a square lattice

As expected, our analysis did not produce an AF insulator in the Hubbard model without the explicit AF spin-exchange necessary to generate it. Therefore we use the $t - U - J$ model to search for a long range spin-ordered state. It is interesting to note that an AF spin coupling $J = t/4$ causes a discontinuous transition from a uniform metallic state to an AF ordered insulating state in the case of a square lattice^{23,24} in the absence of field. The metallic state has a non-zero staggered magnetization (m), decreasing as the AF spin exchange decreases.

In this context, we may note that the picture is quite different for a triangular lattice: a magnetic order (the classical Néel state) and a sudden drop in QP weight appear at the same point (Fig. 6)²⁴. The triangular lattice has no nesting, no particle-hole symmetry at half-filling and mitigates staggered magnetization in the metallic state. The square density of states, on the other hand, has a logarithmic divergence at zero energy and is amenable to an AF order at half-filling. For the latter a non-zero next nearest neighbor hopping (t'), therefore, introduces the necessary frustration, as opposed to the inherent geometric one in a triangular lattice. The same sign of t' as t ensures a convex Fermi surface for the spinon sector.

The role of external field is to destroy the staggered magnetization and favor an unsaturated ferromagnetic metal and finally a saturated ferromagnetic insulating state in the $t - U - J$ model on a square lattice. However, as we discussed above the $t - t' - U - J$ model would be the natural choice for a square lattice, in which, the long-range order due to nesting is suppressed by the frustration of the interactions. In the next section we discuss the MIT in the $t - t' - U - J$ model and the effect of Zeeman field on it.

C. $t - t' - U - J$ model on a square lattice

The single-site analysis of $t - t' - U - J$ model on the square lattice shows a first order MIT at $U_c = 5.131t$ in the absence of magnetic field. Switching over to a two-

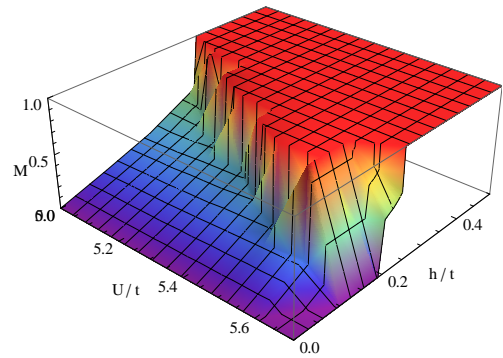


FIG. 7. $t - t' - U - J$ model on a square lattice ($t = 1.0$, $t' = 0.50$, $J = 0.25$): A close-up view of MMT in the cluster analysis very near to the MIT. The metallic side is qualitatively similar to Fig. 2, while the insulating side has two distinct jumps (see text).

site cluster drives the critical correlation to a larger value $5.653t$. In the single-site case the staggered magnetization saturates at the MIT and in the insulating side the spin and charge dynamics get quenched completely. In the cluster extension, however, the insulating state still has non-local phase fluctuations that lead to a finite spin stiffness and therefore the staggered magnetization never saturates. The finiteness of B for large values of correlation implies that the AF order does not saturate. Hence, in the insulating side, on application of external magnetic field, a minimum $h = J$ is needed to destroy the AF order and revive ferromagnetism in the single-site theory. The metallic side of the problem in the single-site theory remains featureless except that there is no AF metal now. A strong local correlation very close to MIT, however, competes against this frustration and produces a weakly AF metal.

The cluster analysis reveals interesting physics in both the metallic and insulating regimes. While the MIT becomes second order in this case, the Zeeman field competes with the exchange energy J as well as the emergent super-exchange scale in the insulating side. However, the MMT for all finite values of local correlation still survives for both site and cluster analysis. The $M - U - h$ phase diagram (Fig. 7) from the cluster analysis of the present model, zoomed around the MIT region, reveals interesting details beyond the critical local correlation. There are two distinct jumps in the magnetization for $U = 5.7t > U_c = 5.653t$. These two manifestly separate locations of the jump correspond to the competition between two different scales with the external Zeeman field: the first one is where antiferromagnetism gets suppressed and ferromagnetism shows up, and the second one is where the spins are ferromagnetically saturated at an energy scale given by the residual kinetic energy ($Bt \sim t^2/U$).

IV. EXPERIMENTAL REALIZATIONS

Metamagnetism has, of late, become a highly recurrent^{7,9-13,27} phenomenon. The perpendicular or in-plane field required for such discontinuous, super-linear transitions in magnetization can be under 10 Tesla. All the strongly correlated systems mentioned earlier show MMT between 1 to 10 Tesla fields. However, the MM in each of these is material-specific and often associated with some structural transitions. The role of correlation is not clear in most of them, while our concern is MM out of correlation alone where the competition between the applied field and the local spin fluctuation is the key in driving the non-linear magnetization and the consequent first order jump.

On the long-standing issue of MM in liquid He³, Georges and Laloux¹⁹ hold the view that liquid He³ should be viewed as a Mott-Stoner liquid and Hubbard model with about 8% vacancy offers a reasonable description of it. They predict MMT at about 26 bar in 80 Tesla field. Weigers¹⁴, however, did not find a metamagnetic jump in liquid He³ up to 200 Tesla. This may indicate¹⁹ that He³ cannot be modelled by a half-filled Hubbard model. Vollhardt⁵ puts liquid He³ in the intermediate coupling regime with U/E_F in the range less than one (typically 0.8 or less). As we have shown above, the fields required for MMT is amenable only in the strong coupling regime and therefore Vollhardt's estimates would imply that liquid He³ is not an ideal candidate for the observation of correlation-driven MMT within an accessible laboratory field.

The problem, therefore, is to find a material that can be modelled well by Hubbard model or any of its extended incarnations with desired range of parameters. In correlated systems the bare value of $k_B T_F$ (T_F is Fermi temperature) is nearly 1-5 eV, which (T_F) scales down to $k_B T_K = Zt^*$ close to Mott MIT. Exactly at this range of parameters, as we showed, h_c for MMT becomes $< 100T$. The conclusion is driven by the fact that for $t = 1eV$, in the site analysis, we get $h_c/t < 0.01$, which is equivalent

to a field of 100 Tesla or less. We find that for a system with U/t of order $0.99U_c$, the typical value of critical Zeeman field, by a similar analysis, would be about 20 Tesla ($h_c/t < 0.002$). A strongly correlated system which can be reasonably modelled by single band Hubbard model, with its effective correlation U/U_c tuned (by pressure, for example) somewhere between 0.97 to 0.99, should, therefore, show an MMT within a reasonable magnetic field. It is also likely that cold atom systems in optical lattices should provide us the option of observing this in the laboratory. The scaling analysis we discuss here, therefore, underlines the fact that systems with narrow correlated bands or orbital selectivity (in multi-orbital situations, for example) can facilitate MMT at an accessible field.

V. CONCLUSIONS

A ground state analysis of the Hubbard model and its spin-correlated, spin-frustrated versions have been performed in the present paper in search of MM. The response to an externally applied Zeeman field in the SR mean field formalism shows that there indeed is a regime of parameters where an MMT to a ferromagnetic state occurs for all finite local Hubbard correlations. However, it is clear that to observe MM in a laboratory field, one would need to tune the system to a very narrow range close to Mott transition and apply a fairly large magnetic field. These conditions make the observation of MM in such systems so elusive. On a fundamental level, on the other hand, one would like to know what happens to the spin-fluctuation scale and the Kondo scale in a metal in the presence of a strong field favoring spin-alignment. Clearly, it is the sharpest Kondo resonance peak, with narrowest width (requiring close proximity to Mott transition), that is most sensitive to external field and leads to non-linear jump in magnetization and the consequent MMT.

Acknowledgement: The authors thank Vijay B. Shenoy for discussions and comments on the manuscript. S.A thanks Serge Florens for useful interactions at the initial stage and UGC (India) for a fellowship.

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